THE MISSING TORQUE

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Abstract

In this work we present a very simple situation where the missing torque problem arises. Then we show that the reasonings utilized recently by some authors to solve the problem are incorrect.

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1 - Introduction

In recent years some authors have been worried about a problem that, at first glance, seems easy to be solved: the dynamics of moving magnetic dipoles, [1] to [3].

The system consists of a magnetic dipole (a current loop) and a straight charged infinite wire with current. Considering special positions between them, for instance the dipole moment parallel or orthogonal to the wire, they analyse the system in two different inertial frames: the laboratory one, with the loop at rest, and a frame where there is no current in the wire, with the loop moving. Then it is seen that in the second frame there is a lack of a torque to counterbalance the magnetic one. In the case where the magnetic moment of the loop is perpendicular to the wire, all the authors agree that the physical origin of the missing torque is the relativity of simultaneity. On the other hand, in the case where the magnetic moment is parallel to the wire, each author presents a different origin to the missing torque.

In the next section we present a simpler situation where the missing torque problem arises. We analyse the problem and we show later what are the mistakes made by the authors of references [1] to [3].

2 - The Missing Torque Problem

Suppose an infinite and straight charged wire near a neutral magnetic dipole. Both are at rest relative to one another and to the inertial frame S. The magnetic moment of the loop is parallel to the wire. The sides of the loop are in the radial and azimuthal directions, as shown in Figure 1. In the frame S the wire generates only an electric field given by:

$$\vec{E} = \frac{\lambda \hat{\rho}}{2\pi\epsilon_0 \rho} \ , \tag{1}$$

where λ is the linear charge density of the wire, ρ is the radial distance to the wire and $\hat{\rho} = \vec{\rho}/\rho$ is the radial unit vector.

We now analyse the same situation from another inertial frame S' that is moving with velocity $\vec{v} = -v\hat{z}$ relative to S (Figure 2). Relative to this frame S' there is an electric current in the wire that generates a magnetic field $\vec{B}$. Due to the Lorentz contraction, the linear charge density of the wire in the frame S' is given by $\lambda' = \lambda/(1 - v^2/c^2)^{1/2}$. So the electric field generated by the wire in the frame S' is given by $\vec{E'}$ instead of $\vec{E}$. These fields are given by:

$$\vec{E'} = \frac{\lambda' \hat{\rho}}{2\pi\epsilon_0 \rho} \ , \tag{2}$$

$$\vec{B'} = -\vec{v} \times \frac{\vec{E'}}{c^2} = \frac{\mu_0 I' \hat{\theta}}{2\pi\rho} \ , \tag{3}$$

where

$$\lambda' = \gamma\lambda = \frac{\lambda}{(1 - v^2/c^2)^{1/2}} \ , \tag{4}$$

$$I' = \gamma\lambda v \ . \tag{5}$$

Here the problem arises. It is clear that in frame S there is no torque in the loop. But in S' there is a net torque acting on the dipole. The torque can be seen analysing the force on any electron in the side bc of the loop (see Figure 2). The Lorentz force $\vec{F'}$ acting on the charge $q = -e$ is given by:

$$\vec{F'} = -e(\vec{E'} + \vec{v'} \times \vec{B'}), \tag{6}$$

where $\vec{v'}$ is the total velocity of the electron relative to S' (see reference [4]). Observing that $\vec{v'} = v_d \hat{\rho} + v\hat{z}$, where v_d is the drifting velocity, and using (2) and (3) yields:

$$\vec{F'} = -e(\frac{\lambda'}{2\pi\epsilon_0\rho} - \frac{\mu_0 I' v}{2\pi\rho})\hat{\rho} - \frac{e\mu_0 I' v_d \hat{z}}{2\pi\rho}. \tag{7}$$

The second term in the right hand side of equation (7) is what generates the torque on the loop. The reason is that if we calculate the force on an electron in the side ad, the z-component of the force will be in the opposite direction. There should be an opposite torque to balance this one. But its physical origin is not easy to find. This is the missing torque problem.

3 - Proposed Solutions

To solve this problem, the authors in [1] – [3] proposed different approaches, but all of them have a common assumption: they utilize *internal* *forces* in the loop to counteract the force in the direction z generated by the magnetic field. This is the flaw of their solutions because *an internal force can not induce the dynamics of a system.* So it can not be the explanation of the missing torque. This is a direct consequence of Newton's third law (even in the framework of special relativity all the authors of references [1] to [3] correctly agree that Newton's third law should be valid in this geometry when the magnetic moment of the loop is parallel to the wire).

Let us see the particular case of reference [1]. According to them, beyond the force given by equation (7), there is another force exerted by the "wall" of the loop on the electron. They estimate it using the relativistic expression of force and accepting that there is no acceleration of the charge in the z direction. The magnitude of this force in the z direction is equal to the second term of equation (7) and, using the law of action and reaction, they argue that the electron will exert the same force in the opposite direction on the "wall," annuling the torque. This is incorrect in our conception because the total force in the z - direction on the side bc (or ad) of the loop is given by:

$$F^z_{loop} = F^z_{B\ on\ q} + F^z_{B\ on\ w} + F^z_{q\ on\ w} + F^z_{w\ on\ q}\ , \tag{8}$$

where the subscripts B, q and w represent, respectively, the magnetic field, the electron and the "wall".

We know that $F^z_{B\ on\ w} = 0$ and that $F^z_{q\ on\ w} = -F^z_{w\ on\ q}$, so that

$$F^z_{loop} = F^z_{B\ on\ q} = \frac{-\mu_0 v_d I'}{2\pi\rho}\ . \tag{9}$$

As we can see in equation (9), there is still a net torque on the loop. Invoking the "wall" does not solve the problem.

Namias, in his paper [2], proposes an equation for the torque given by:

$$\vec{\tau} = \vec{m} \times \vec{B} - (\vec{m} \times \vec{v}) \times \frac{\vec{E}}{c^2} - \vec{v} \times [\vec{m} \times (\frac{\vec{E}}{c^2})]\ . \tag{10}$$

where $\vec{m}$ is the magnetic moment of the loop.

The last term of equation (10) would be the missing torque, whose phy– sical explanation would be internal forces generated by induced charges on the dipole.

He does not prove equation (10), obtaining it from an analogy with an equation for an electric dipole and using the model of magnetic charges for the magnetic dipole. To us this is not a real proof. The analogy between magnetic charges and magnetic dipoles is not always valid, as was shown by

Boyer in [5]. We have also seen reference [6] (mentioned by Namias when presenting equation (10)) and there is nothing there about equation (10). So we think that his solution is not convincing.

Spavieri in [3] suggests a stress-energy tensor that, under an external electric field, would generate an internal torque. This idea generates the same problem of internal forces like the paper [1] of Bedford and Krumm. As we have said, all of them utilize essentially the same idea, *internal forces*, and this can not be a solution of the problem.

4 - Conclusion

We have seen in this paper a very simple situation where, using the ordinary field transformations between two inertial frames, a non-vanishing torque arises in one frame of reference but not in another one. Although it is simple, this problem is very important because it is well-grounded in basic ideas of electromagnetism. So it should have a solution. Regarding this problem, some authors have already proposed solutions, but we have also shown that they have made a mistake when considering that internal forces could cancel external torques.

We do not know a solution for this simple problem utilizing the framework of relativity. Our only goal here was to point out flaws in the published solutions of this problem which arised up to now.

Figure Captions

Fig. 1 – An infinite rectilinear wire (along the z axis) at rest in an inertial frame S, charged with a constant linear charge density λ. Also at rest in this frame is a neutral electric circuit *abcd* where flows the constant current I. Its magnetic moment is parallel to the wire.

The pieces *bc* and *da* are radial segments of current, while *ab* and *cd* are segments of the circuit along arcs of circle centered on the z axis.

Fig. 2 – The same system seen from the inertial frame S' which moves relative to S with a constant velocity $-v\hat{z}$.

References

[1] Bedford, D. and Krumm, P., " On the origin of magnetic dynamics," American Journal of Physics, volume **54**, pp. 1036 – 1039 (1986).

[2] Namias, V., "Electrodynamics of moving dipoles: The case of the missing torque," American Journal of Physics, volume **57**, pp. 171–179 (1989).

[3] Spavieri, G., "Proposal for experiments to detect the missing torque in special relativity," Foundations of Physics Letters, volume **3**, pp. 291–301 (1990).

[4] Assis, A. K. T. and Peixoto, F. M., "On the velocity in Lorentz force law," The Physics Teacher, volume **30**, pp. 480 – 483 (1992).

[5] Boyer, T. H., "The force on a magnetic dipole," American Journal of Physics, volume **56**, pp. 688 – 692 (1988).

[6] Fano, R. M., Lan, J. C. and Adler, R. B., *Electromagnetic Fields, Energies and Forces* (Wiley, New York, 1960), pp. 271 – 275.