

Increasing the number of atoms in a MOT with transverse 2D cooling

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Using a frequency spread light and a zigzag beam configuration, an atomic beam of ${}^7\text{Li}$ has been transversally cooled and compressed in 2D. Our objective was to increase the loading rate of our magneto-optical trap (MOT) to capture a large number of atoms. We have gained a factor of 10 to a total of 2×10^9 atoms. We analyze the loading rate dependence on the 2D beam intensity and propose a simple phenomenological model that explains our results.

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I. INTRODUCTION

The idea of using transverse cooling to collimate an atomic beam was proposed for the first time in the classical paper of Hänsch and Schawlow [1]. Since then it has been used by many groups whenever a large beam brightness or a small beam divergence is desired [2–11]. Transverse cooling can be performed either along one (1D) or two (2D) axis perpendicular to the atomic beam. It is usually performed with a monochromatic standing wave [3–6, 8, 10, 11], using a curved wavefront [2, 12] or a zigzag configuration [7, 13]. A recent article [9] compared all these methods on a beam of metastable helium and concluded that a multi-frequency light in a zigzag configuration gives the largest compression.

In our experiment we have employed frequency spread light in a zigzag configuration to transversally 2D cool a beam of ${}^7\text{Li}$. Our objective was to increase the beam brightness to improve the loading rate of our MOT. We have gained a factor of 13 in loading rate, allowing us to trap 2×10^9 atoms.

In section II we present our experimental setup for the MOT and 2D cooling and the necessary condition for maximum compression. We show our results and the loading rate dependence with beam intensity and the number of reflections on the mirror. In section III we use Doppler cooling theory to model the interaction of the atoms with the 2D light. A phenomenological model is proposed in section IV that explains the gain in loading rate. Using the Fokker-Planck equation we are able to fit the loading rate dependence with beam intensity by adjusting the saturation intensity. In section V we present our conclusion.

II. EXPERIMENTAL SETUP AND RESULTS

Figure 1 shows the configuration of our experiment. The ${}^7\text{Li}$ atoms leave the oven at $T \cong 320^\circ\text{C}$ and are

transversally cooled inside a 15 cm^3 cube. Four mirrors have been used, each one 9 cm long, paired in two orthogonal axis. A fraction of the MOT light (60 mW) was used for the 2D beams. After being cooled, the atoms pass through a 16 cm long skimmer that separates the cube from the trapping chamber. Our MOT is of the standard design with a radial gradient of 21 G/cm. There are three retro-reflecting frequency spread beams (comb light) with a peak intensity of 75 mW/cm^2 each and a $1/e^2$ diameter of 14.8 mm. The frequency of the dye laser is locked to the $2S_{1/2}, F=2 \rightarrow 2P_{3/2}, F=3$ ${}^7\text{Li}$ transition (671 nm) using a saturated absorption cell. The comb light was generated with a double pass through an electro-optic modulator (EOM) at 11.7 MHz. The frequency of the light is spread from -127.5 MHz (-5th order) to -10.5 MHz (+5th order) bellow the locked transition. For more information on this technique see reference [14].

Figure 2 shows the trajectory of the 2D beams and the atoms. The beams are injected with an angle of 87.5° with the symmetry axis z . Not shown are the two other ones that are perpendicular to the figure. We have aligned the beams and tilted the mirrors in order to bounce back and forth the light in a zigzag configuration, covering the whole mirror. This way the atoms had a long interaction length, maximizing their compression. The light is linear polarized with an intensity of 63 mW/cm^2 and a $1/e^2$ diameter of 7.78 mm.

The compression of the atomic beam is limited by the Doppler temperature. This is the minimum temperature that a two-level atom can reach through radiation scattering [15]. Even though ${}^7\text{Li}$ has more than two levels, this is a good approximation for our experiment, since lithium does not have sub-Doppler cooling. The atoms reach this limit exponentially with a $1/e$ decay time τ_d that is a function of detuning and intensity of the cooling beam (see section III). The mirror length had been chosen in order that the atoms had enough time to reach the Doppler limit. This condition can be expressed as $\tau_d \ll T$, where T is the interaction time of the fastest atom captured by the MOT. The velocity of this atom is v_c (the capture velocity of the MOT), giving the following condition:

$$\frac{\tau_d v_c}{L} \ll 1, \quad (1)$$

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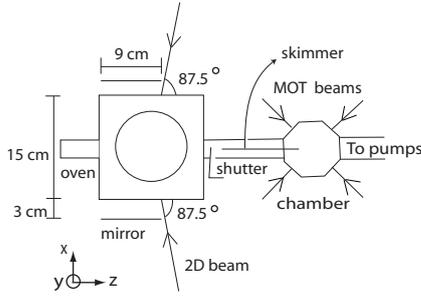


FIG. 1: This is our experimental setup. Not shown in the picture are the two mirrors perpendicular to the y direction and the third MOT beam perpendicular to the xz plane. The shutter is closed after the trap is loaded. The chamber is kept at ultra-high vacuum (lifetime = 10 s).

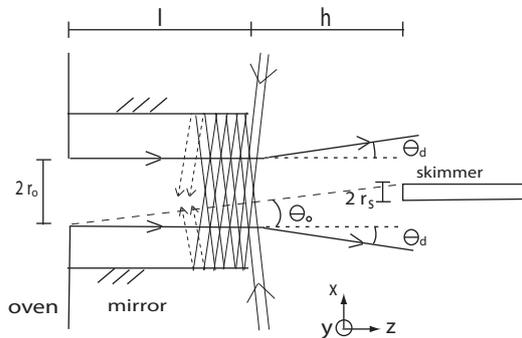


FIG. 2: The 2D beams have a zigzag trajectory covering the entire mirror length l . After the cooling the atoms still have a transverse expansion due to the Doppler speed. The angle θ_d is given by v_d/v_l , where v_d is the Doppler speed and v_l is the longitudinal velocity after cooling. The distance h is 124 mm, the oven radius r_o is 6.35 mm and the skimmer radius r_s is 2.50 mm.

where $L = l + D$ is the interaction length, l is the mirror length, and D is the beam diameter. Using the same method described in [14], we have estimated $v_c = 127$ m/s. The time τ_d can be calculated as being $\tau_d = 285 \mu\text{s}$ (see section III). The interaction length is $L = 10$ cm, so we have $\tau_d v_c / L = 0.36 < 1$. That means that the atoms reach transverse Doppler speed after the first 4 cm inside the cube.

A photomultiplier tube was used to measure the loading rate. In the first 3 s its behavior is practically linear and we can use $R = \Delta N / \Delta t$. The absolute number of atoms was given by the fluorescence measured with a calibrated CCD camera. The measured value of the gain in loading rate is $R'/R = 13 \pm 1$, where R' is the loading rate with 2D cooling. The absolute value of R' is $2 \times 10^8 \text{ s}^{-1}$, giving 2×10^9 atoms in the MOT after loading. The 2D cooling is very robust regarding the beam alignment. It is only sensitive to the power divi-

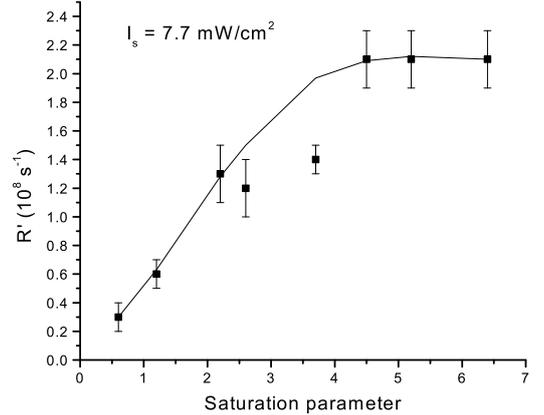


FIG. 3: The saturation parameter is given by I/I_s . The saturation intensity I_s is calculated from the fit using the χ^2 method. For high intensities the loading rate saturates. At this level most of the atoms have reached Doppler temperature in the transverse direction. The fit is given by the model shown in section IV.

sion among the four beams.

Figure 3 shows the dependence of R' with the 2D beam intensity when we change the beam power. The fit was obtained with the phenomenological model presented in section IV. At high intensities R' saturates as an indication that the atoms have reached the Doppler limit. At low intensities it approaches the value with no cooling.

We have also changed the number of reflections M on each mirror to see how R'/R varies (Fig. 4). The value $M = 10$ corresponds to maximum interaction time while $M = 0$ is the configuration where there are only two retro-reflected beams orthogonal to the atom trajectory. The interaction length is proportional to the number of reflections. Figure 4 shows that for $M \leq 5$ the loading rate begins to decrease very fast. This is because the atoms do not interact with the light for enough time to reach the Doppler temperature, limiting its compression. When we have only a standing wave ($M = 0$), the gain is almost none ($R'/R = 1.6 \pm 0.1$). This shows that a long interaction length that satisfies (1) is necessary if we want maximum beam compression.

III. DOPPLER COOLING

The Doppler cooling theory is the simplest one that can be used to study the atom-light interaction. It assumes a 2-level atom and it treats multiple laser beams independently [15, 16]. In some cases this approximation is too simplified and then you have to take into account other atomic levels, the polarization of the beams and the coherent interaction between the atom and the light [17–27]. In our experiment, since the 2D beams are travelling waves and there is no magnetic field, the Doppler

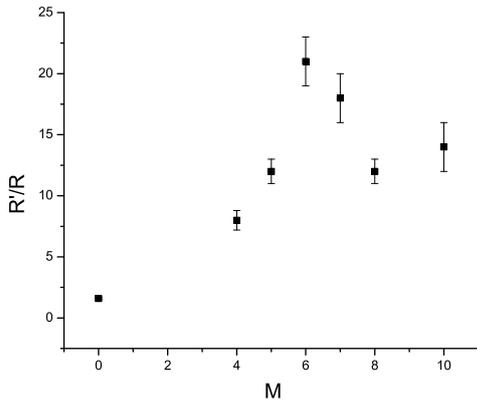


FIG. 4: A lower number of reflections is equivalent to a smaller interaction length. At $M = 0$ we see that we have almost no gain in loading rate. The larger values around $M = 6$ are caused by fluctuations on the laser beam power from day-to-day measurements.

theory is a good approximation. The scattering force on a 2-level atom by a monochromatic plane wave is given by [16]:

$$\vec{F} = \frac{\hbar\Gamma\vec{k}}{2} \left[\frac{S}{1+S+(2\Delta/\Gamma)^2} \right], \quad (2)$$

where Γ is the spontaneous emission rate ($\Gamma = 2\pi \times 5.9$ MHz), \vec{k} is the wave vector, $S = I/I_s$ is the saturation parameter and Δ is the detuning. The comb light is composed of many frequencies and in order to simplify our model, we will treat them independently. The total force that the 2D beams exert on an atom is given by [14]:

$$F_t = \frac{\hbar\Gamma k S}{2} \sin\eta \sum_{i=-5}^5 f_i \left[\frac{1}{1+S+(2\Delta_{2i}/\Gamma)^2} - \frac{1}{1+S+(2\Delta_{1i}/\Gamma)^2} \right], \quad (3)$$

$$F_l = -\frac{\hbar\Gamma k S}{2} \cos\eta \sum_{i=-5}^5 f_i \left[\frac{1}{1+S+(2\Delta_{1i}/\Gamma)^2} + \frac{1}{1+S+(2\Delta_{2i}/\Gamma)^2} + \frac{2}{1+S+(2\Delta_{3i}/\Gamma)^2} \right], \quad (4)$$

$$f_i = \{11.7\%, 5\%, 9.6\%, 15.8\%, 10\%, 3.75\%\},$$

$$f_{-i} = f_i, \quad i \in \{0, \dots, 5\}, \quad (5)$$

$$\delta_i = 2\pi \times (11.7i - 68.6)\text{MHz}, \quad (6)$$

$$\Delta_{1i} = \delta_i + kv_t \sin\eta + kv_l \cos\eta, \quad (7)$$

$$\Delta_{2i} = \delta_i - kv_t \sin\eta + kv_l \cos\eta, \quad (8)$$

$$\Delta_{3i} = \delta_i + kv_l \cos\eta, \quad (9)$$

where the sum is over all comb sidebands, η is the beam angle with the atomic beam (87.5°) and f_i and δ_i are the power fraction and detuning of each sideband. The transverse force F_t is responsible for the beam compression and F_l is the longitudinal cooling force. Because the 2D beams are almost perpendicular to the atomic beam, the compression is the main effect observed.

In the low velocity limit, equation (3) can be expressed as [15]:

$$\vec{F}_t = -\alpha\vec{v}, \quad (10)$$

$$\alpha = -4\hbar k^2 S \sin\eta \sum_i \frac{f_i(2\delta_i/\Gamma)}{[1+S+(2\delta_i/\Gamma)^2]^2}. \quad (11)$$

Equation (10) suggests that v exponentially decays to zero. However this does not happen because the atom is constantly heated up by the recoil due to absorption and spontaneous emission [15, 16]. It drifts in momentum

space with a diffusion constant given by [15]:

$$D_p = \hbar^2 k^2 \gamma, \quad (12)$$

$$\gamma = \frac{\Gamma}{2} \sum_i \frac{f_i S}{1+S+(2\delta_i/\Gamma)^2}, \quad (13)$$

where γ is the total photon scattering rate. The result is that the atoms reach a terminal velocity, called the Doppler speed v_d , with a decay time τ_d [15]:

$$\tau_d = m/\alpha, \quad (14)$$

$$v_d = \frac{\hbar k}{\sqrt{m}} \sqrt{\gamma/\alpha}, \quad (15)$$

where m is the mass of the atom. For $S = 7.6$ (maximum intensity on the 2D beams), equations (11), (13), (14) and (15) give $\tau_d = 285 \mu\text{s}$ and $v_d = 1.19$ m/s.

IV. PHENOMENOLOGICAL MODEL

A rigorous theoretical study of the 2D cooling would be very complicated. We will show that using Eqs. (3)-(15) and some simple physical assumptions, we can make a phenomenological model that explains very well our results. First we will model the gain in loading rate and later its dependence on the intensity of the 2D beams.

The atoms leaving the oven follow an effusion process. This is because the oven aperture is much smaller than the mean-free path p of the atoms in the cube ($r_{oven} = 6.35 \text{ mm} \ll p \sim 10^6 \text{ mm}$). In this process the velocity distribution of the atoms per unit area, per unit time in spherical coordinates is [28]:

$$\Phi(\vec{v})d^3v = n \left(\frac{m}{2\pi k_B T} \right)^{3/2} v \cos \theta e^{-mv^2/2k_B T} d^3v, \quad (16)$$

where n is the atomic beam density, T is the oven temperature, k_B is the Boltzmann constant and θ is the azimuthal angle. Knowing that the MOT can only capture atoms up to a certain velocity v_c and assuming $v_c \ll \sqrt{2k_B T/m}$, we can integrate (16) and obtain an approximate expression for the loading rate:

$$R \propto n v_c^4. \quad (17)$$

Using eq. (17) the gain in loading rate can be expressed as:

$$\frac{R'}{R} = \left(\frac{n'}{n} \right) \left(\frac{v'_c}{v_c} \right)^4 \beta, \quad (18)$$

where the prime sign means that the parameter is measured with 2D cooling. As we will see later $v'_c > v_c$. The reason is that when there is 2D cooling, atoms with initial velocity larger than v_c can be cooled down and captured by the MOT. The factor β takes into account the difference in size between the atomic beam after cooling and the skimmer aperture. It is given by [29]:

$$\beta = \left(\frac{h \theta_d + r_s}{r_o} \right)^2, \quad (19)$$

where h is the distance between the last interaction spot and the skimmer, r_s and r_o are the skimmer and the oven aperture radius and θ_d is the divergence angle of the atomic beam after 2D cooling (see Fig. 2). Since the beams are almost perpendicular to the atoms ($\eta = 87.5^\circ$), the main effect of the 2D cooling is the increase in the atomic beam density. Assuming the number of atoms is conserved, the gain is given by [29]:

$$\frac{n'}{n} = \left(\frac{v_{ct}}{v'_t} \right)^2 \left(\frac{v_c}{v'_l} \right), \quad (20)$$

where v_{ct} is the transverse capture velocity and v'_t and v'_l are the atomic transverse and longitudinal velocities after 2D cooling. Using (20) in (18):

$$\frac{R'}{R} = \left(\frac{v_{ct}}{v'_t} \right)^2 \left(\frac{v_c}{v'_l} \right) \left(\frac{v'_c}{v_c} \right)^4 \beta. \quad (21)$$

It is now necessary to calculate the value of each parameter in Eq. (21). The MOT capture velocity v_c is estimated using Eq. (3) with $\eta = 45^\circ$, $I_s = 7.7 \text{ mW/cm}^2$ and $S = 12$. We integrate (3) and calculate

the maximum initial velocity an atom can have before it stops inside the MOT [14, 30]. The value of v_{ct} is fixed by the oven-skimmer geometry: $v_{ct} = v_c \tan \theta_0$, where $\tan \theta_0 = (r_o + r_s)/(L + h) = 3.8 \times 10^{-2}$ (see Fig. 2) [31]. The transverse velocity v'_t depends on the 2D beam intensity and the interaction length L . Considering that Eq. (1) is satisfied at full power, we can approximate $v'_t \cong v_d$, where v_d is the Doppler speed given by (15) [29]. The remaining two terms, v'_l and v'_c are calculated using Eq. 4 with the following conditions: $v'_l = v_l(L)$ assuming $v_l(0) = v_c$ and $v'_c = v_l(0)$ given $v_l(L) = v_c$. The values are:

$$v_c = 127 \text{ m/s}, \quad (22a)$$

$$v_{ct} = 4.83 \text{ m/s}, \quad (22b)$$

$$v'_t = 1.19 \text{ m/s}, \quad (22c)$$

$$v'_l = 122 \text{ m/s}, \quad (22d)$$

$$v'_c = 131 \text{ m/s}, \quad (22e)$$

$$\beta = 0.34. \quad (22f)$$

Applying Eqs. (22) in (21) we get $(R'/R)_{theory} = 7$. This is close to the experimental result 13 ± 1 . Considering all the approximations that we have made to estimate Eqs. (22), it is not a surprise that we did not get $(R'/R)_{theory}$ within the experimental error of $(R'/R)_{exp}$.

The values (22d) and (22e) are very similar to (22a). This supports what we have said before that there is very little cooling in the longitudinal direction. Hence a good approximation of Eq. (21) is:

$$\frac{R'}{R} \cong \left(\frac{v_{ct}}{v'_t} \right)^2 \beta. \quad (23)$$

In order to understand the dependence of R' with intensity (Fig. 3), we will focus on Eq. (23). When the 2D beam intensity varies, v_{ct} is a constant and β varies very little. So we have $R' \propto 1/(v'_t)^2$. Let us now consider two extreme cases. When $I = 0$, $v'_t = v_{ct}$ and $R' = R$. If I is very large ($S \gg 1$), then v'_t saturates at the Doppler speed. These limits seem to agree with the graph in Fig. 3, therefore if we could obtain an equation for v'_t , we would be able to fit our data. However there is not an analytical formula for v'_t , because the spontaneous emission introduces a stochastic term in Eq. (3). The proper way to study this problem is to analyze the evolution of the velocity distribution using a Fokker-Planck equation. In the case of laser cooling it is given by [32–34]:

$$\frac{\partial \rho}{\partial t}(v, t) = \frac{\partial}{\partial v} \left[\frac{D_p}{m^2} \frac{\partial \rho}{\partial v} - \frac{\rho}{m} F(v) \right], \quad (24)$$

where $\rho(v, t)$ is the atomic velocity distribution, D_p is the momentum drift coefficient and $F(v)$ is the velocity-dependent force. We can now use Eq.(24) to obtain a phenomenological model for the average $\langle (v'_t)^2 \rangle$. In the demonstration we will simplify the notation by defining $v'_t \equiv v$. We begin with the average $\langle v^2 \rangle$ for an ensemble

($\int_{-\infty}^{\infty} \rho dv \equiv 1$):

$$\langle v^2 \rangle(t) = \int_{-\infty}^{\infty} v^2 \rho(v, t) dv. \quad (25)$$

Taking the derivative of Eq.(25) we get:

$$\frac{d\langle v^2 \rangle}{dt} = \int v^2 \frac{\partial \rho}{\partial t}(v, t) dv, \quad (26)$$

Using (24) in (26) and integrating by parts:

$$\frac{d\langle v^2 \rangle}{dt} = -\frac{2D_p}{m} \int v \frac{\partial \rho}{\partial v}(v, t) dv + \frac{2}{m} \int v \rho(v, t) F(v) dv. \quad (27)$$

Applying (10), (14) and (25) in (27):

$$\frac{d\langle v^2 \rangle}{dt} = -\frac{2}{\tau_d} \langle v^2 \rangle - \frac{2D_p}{m^2} \int v \frac{\partial \rho}{\partial v}(v, t) dv. \quad (28)$$

Integrating by parts once again (28):

$$\frac{d\langle v^2 \rangle}{dt} = -\frac{2}{\tau_d} \langle v^2 \rangle + \frac{2D_p}{m^2}. \quad (29)$$

Using (12), (14) and (15) in (29):

$$\frac{d\langle v^2 \rangle}{dt} = -\frac{2}{\tau_d} \langle v^2 \rangle + \frac{2}{\tau_d} v_d^2. \quad (30)$$

Equation (30) has a simple physical interpretation. The first term on the right hand side is due to the linear cooling force (10) and the second term is due to a constant heating that simulates the effect of the spontaneous emission recoil. Its solution for an atom at $t = T$ and 2D cooling intensity I is:

$$\langle (v'_t)^2 \rangle(I) = v_d^2(I) + [v_{ct}^2 - v_d^2(I)] e^{-2T/\tau_d(I)}, \quad (31)$$

where $T = L/v_c$ is the interaction time and the initial condition is $\langle (v'_t)^2 \rangle = v_{ct}^2$ for $T = 0$. Using (14), (15)

and (22b) in (31), we can fit Fig. 3 and estimate the saturation intensity I_s using the χ^2 -method. The value $I_s = 7.7$ mW/cm² is close to the one previously obtained in [14] (5.2 mW/cm²). In both cases I_s is only an effective saturation parameter, since the 2-level model is an approximation.

V. CONCLUSION

We have successfully implemented transverse 2D cooling to compress an atomic beam of ⁷Li. Using frequency spread light and a zigzag configuration, we increased the loading rate of our MOT by a factor of 13. The advantages of our method are due to the frequency spread light, that increases the capture velocity range, and the long interaction length, that guarantees that the atoms reach the transverse Doppler speed. We developed a theoretical model for the increase in loading rate that agrees within 50% with the experimental value. This is a satisfactory result because we used the simple Doppler theory to obtain the numerical values. In order to model the loading rate dependence with intensity, we used the Fokker-Planck equation to obtain a phenomenological formula for $\langle (v'_t)^2 \rangle$. The model fits very well and it predicts an effective saturation intensity of 7.7. mW/cm² for ⁷Li. In summary transverse 2D cooling with frequency spread light and a zigzag beam configuration is an effective way to compress an atomic beam and to increase a MOT loading rate.

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